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**DAVID W. TAYLOR NAVAL SHIP  
RESEARCH AND DEVELOPMENT CENTER**

Bethesda, Maryland 20084



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COMPRESSIONAL DAMPING IN THREE-LAYER BEAMS  
INCORPORATING NEARLY INCOMPRESSIBLE  
VISCOELASTIC CORES

by

Bruce Edward Douglas



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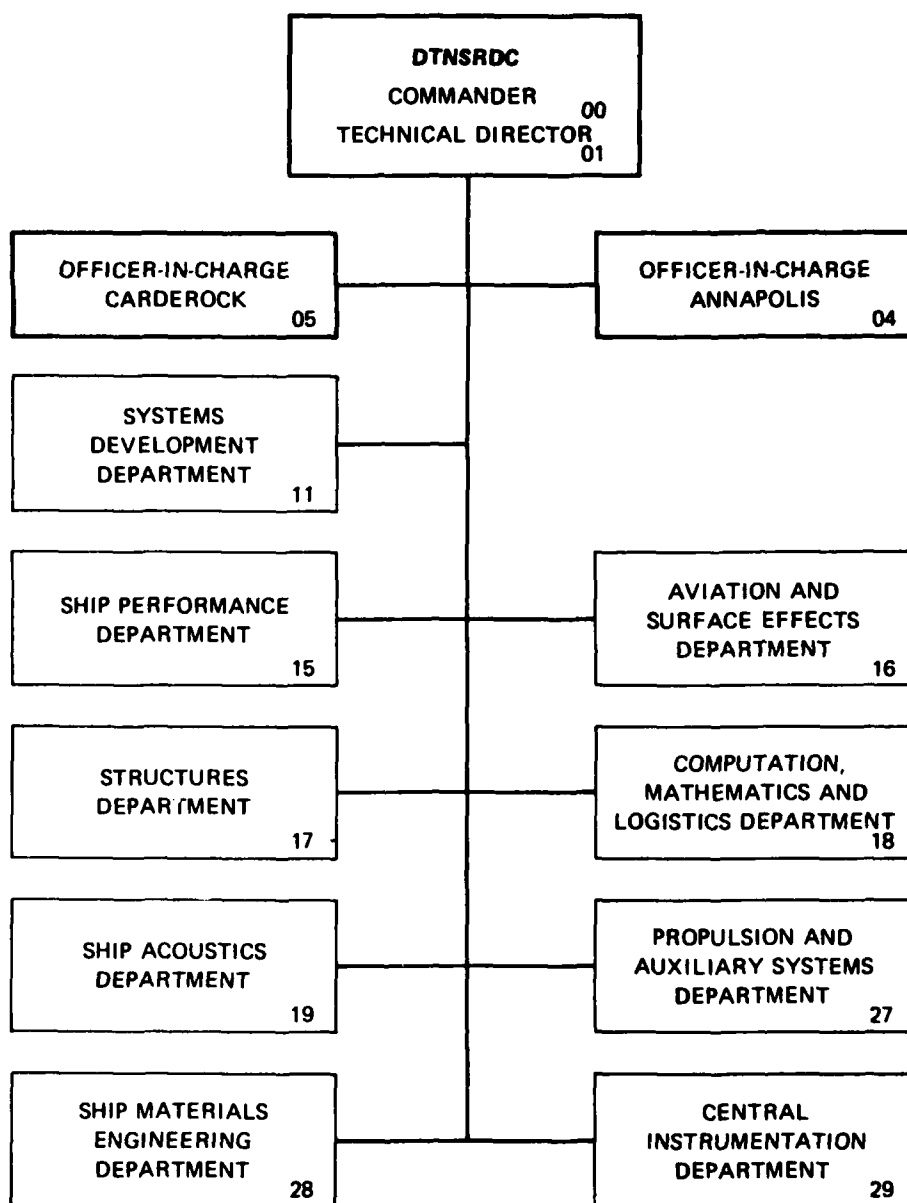
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COMPRESSIONAL DAMPING IN THREE-LAYER BEAMS INCORPORATING NEARLY  
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The transverse free mechanical impedance response of an elastic-viscoelastic-elastic beam incorporating the compressional damping mechanism is considered. The work of Douglas and Yang is extended to include shear deformation and rotary inertia in the elastic layers. The effects of nearly incompressible viscoelastic damping cores on the compressional frequency, and hence on the spectral range of damping effectiveness for the composite,

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is also discussed. Results of the analysis are shown to compare favorably with experimental results for a damped three-layer beam which was optimized for compressional damping and in which the influence of the shear damping mechanism was intentionally minimized.

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# NOMENCLATURE

$b$	Beam width
$E_i$	Elastic (Young's) modulus of <u>i</u> th layer
$E_v$	Elastic storage modulus of viscoelastic core
$E^*(\omega)$	$E_v(1 + i\delta)$ = complex dynamic elastic modulus of viscoelastic core
$G_i$	Shear modulus of <u>i</u> th layer
$I_i$	Moment of inertia of the <u>i</u> th layer
$i$	$\sqrt{-1}$
$k^*$	Compressional spring constant per unit length
$P_o$	Applied force
$r_i$	Radius of gyration for the <u>i</u> th layer
$t$	Time
$t_i$	Thickness of <u>i</u> th layer
$t_v$	Thickness of viscoelastic core
$u(x)$	Longitudinal displacement
$w_i(x)$	Transverse displacement of <u>i</u> th layer
$Z(X,\omega)$	Transverse driving point mechanical impedance of beam at location $x$
$\delta$	Elastic loss tangent of viscoelastic core
$\epsilon_z$	Normal unit elongation (strain) component
$\kappa$	Timoshenko shear constant = 5/6
$\nu_v^*$	Poisson's ratio of viscoelastic core
$\rho_i$	Mass per unit length of the <u>i</u> th layer

$\sigma_z$	Normal stress component
$\omega$	Radial frequency
$\omega_c$	Compressional composite frequency
$l$	Beam length



## ABSTRACT

The transverse free mechanical impedance response of an elastic-viscoelastic-elastic beam incorporating the compressional damping mechanism is considered. The work of Douglas and Yang is extended to include shear deformation and rotary inertia in the elastic layers. The effects of nearly incompressible viscoelastic damping cores on the compressional frequency, and hence on the spectral range of damping effectiveness for the composite, are also discussed. Results of the analysis are shown to compare favorably with experimental results for a damped three-layer beam which was optimized for compressional damping and in which the influence of the shear damping mechanism was intentionally minimized.

## ADMINISTRATIVE INFORMATION

This report represents work performed under the Exploratory Development Acoustical Program, Silencing for Auxiliary Machinery Systems, Program Element 62543N, Task Area SF-43-452-702, Task 18182, Work Unit 2740-112.

The cognizant NAVSEA program manager is Mr. S. G. Wieczorek, NAVSEA (SEA 05H); the DTNSRDC program manager is Dr. Y. F. Wang (Code 2740).

## INTRODUCTION

The need to control the resonance response of naval structures and, as a result, radiated noise has been a prime motivation for research into the dynamic response of inherently damped structures. To date, much of this research has been concentrated on constrained layer damping due to the high composite loss factors attainable. These studies<sup>1-5\*</sup> have, for the most part, emphasized the shear damping mechanism inherent in the composite due to the broad spectral band in which this mechanism could provide high damping performance. Recently, Douglas and Yang<sup>6</sup> examined the compressional damping mechanism inherent in elastic-viscoelastic-elastic beams in the context of the shear damping mechanism. The Mead and Markus model was shown to provide excellent agreement with experimental results for several beams optimized to enhance shear damping. However, this paper also concluded that transverse compressional damping can produce significant dissipation over a relatively narrow spectral band for beams designed to enhance this mechanism.

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\*A complete listing of references is given on page 17.

This report extends the analysis presented by Douglas and Yang<sup>6</sup> to include rotary inertia and shear deformation in the elastic layers. Also, the effects of a nearly incompressible viscoelastic damping core are briefly discussed. The compressional damping mechanism was isolated in this study so that some understanding could be obtained without the complicating influence of other damping mechanisms inherent in the composite.

#### ANALYTICAL DEVELOPMENT

The effects of shear deformation and rotary inertia on the dynamic response of compressionally damped elastic-viscoelastic-elastic beams are significant in the consideration of high order vibrational modes and for low length to thickness ratio beams. For these cases, the equations developed by Douglas and Yang<sup>6</sup> for the fully constrained damped three-layer laminate must be modified by incorporating these terms into the dynamic response of the elastic face layers. By replacing Bernoulli-Euler beam theory with Timoshenko beam theory, the equations of motion for this system can be expressed for steady-state harmonic response as

$$\begin{aligned}
 & \frac{\partial^4 w_1}{\partial x^4} - \frac{\rho_1}{E_1} \left( \frac{1}{\kappa} \frac{E_1}{G_1} + 1 \right) \frac{\partial^4 w_1}{\partial x^2 \partial t^2} + \frac{\rho_1^2}{\kappa E_1 G_1} \frac{\partial^4 w_1}{\partial t^4} \\
 & + \frac{\rho_1}{E_1 r_1^2} \frac{\partial^2 w_1}{\partial t^2} = \frac{k^*}{E_1 I_1} (w_3 - w_1) \\
 & \frac{\partial^4 w_3}{\partial x^4} - \frac{\rho_3}{E_3} \left( \frac{1}{\kappa} \frac{E_3}{G_3} + 1 \right) \frac{\partial^4 w_3}{\partial x^2 \partial t^2} + \frac{\rho_3^2}{\kappa E_3 G_3} \frac{\partial^4 w_3}{\partial t^4} \\
 & + \frac{\rho_3}{E_3 r_3^2} \frac{\partial^2 w_3}{\partial t^2} = \frac{k^*}{E_3 I_3} (w_1 - w_3)
 \end{aligned} \tag{1}$$

where the viscoelastic layer is modeled as a distributed complex compression spring. Assuming a separable solution of the form  $w_1(x,t) = w_1(x)e^{i\omega t}$  and  $w_3(x,t) = w_3(x)e^{i\omega t}$ , Equations (1) can be combined into a single eighth-order differential equation of motion expressed in terms of the response of layer 1

$$\begin{aligned} \frac{d^8 w_1}{dx^8} + (a_1 + b_1) \frac{d^6 w_1}{dx^6} + (a_2 + b_1 a_1 + b_2) \frac{d^4 w_1}{dx^4} \\ + (a_2 b_1 + b_2 a_1) \frac{d^2 w_1}{dx^2} + \left( b_2 a_2 - \frac{k^*^2}{E_1 I_1 E_3 I_3} \right) w_1 = 0 \end{aligned} \quad (2)$$

$$\text{where } a_1 = n_1^4 r_1^2 \left( \frac{E_1}{\kappa G_1} + 1 \right)$$

$$a_2 = \frac{k^*}{E_1 I_1} + n_1^8 r_1^4 \frac{E_1}{\kappa G_1} - n_1^4$$

$$b_1 = n_3^4 r_3^2 \left( \frac{E_3}{\kappa G_3} + 1 \right)$$

$$b_2 = \frac{k^*}{E_3 I_3} + n_3^8 r_3^4 \frac{E_3}{\kappa G_3} - n_3^4$$

$$n_i^2 = \frac{\omega}{r_i} \sqrt{\frac{\rho_i}{E_i}}$$

Employing progressive wave methods, a solution can be written in the form

$$w_1(x) = H_1 e^{\zeta_1 x} + H_2 e^{-\zeta_1 x} + H_3 e^{\zeta_2 x} + H_4 e^{-\zeta_2 x} \\ + H_5 e^{\zeta_3 x} + H_6 e^{-\zeta_3 x} + H_7 e^{\zeta_4 x} + H_8 e^{-\zeta_4 x}$$

and

$$w_3(x) = K_1 \left( H_1 e^{\zeta_1 x} + H_2 e^{-\zeta_1 x} \right) + K_2 \left( H_3 e^{\zeta_2 x} + H_4 e^{-\zeta_2 x} \right) \\ + K_3 \left( H_5 e^{\zeta_3 x} + H_6 e^{-\zeta_3 x} \right) + K_4 \left( H_7 e^{\zeta_4 x} + H_8 e^{-\zeta_4 x} \right) \quad (3)$$

$$\text{where } K_1 = \frac{E_1 I_1}{k^*} \left( \zeta_1^4 + a_1 \zeta_1^2 + a_2 \right)$$

$$K_2 = \frac{E_1 I_1}{k^*} \left( \zeta_2^4 + a_1 \zeta_2^2 + a_2 \right)$$

$$K_3 = \frac{E_1 I_1}{k^*} \left( \zeta_3^4 + a_1 \zeta_3^2 + a_2 \right)$$

and

$$K_4 = \frac{E_1 I_1}{k^*} (\zeta_4^4 + a_1 \zeta_4^2 + a_2)$$

The complex flexural wave numbers  $\zeta_1, \zeta_2, \zeta_3$ , and  $\zeta_4$  can be determined from differential operator methods. Such methods reduce Equation (2) to solving the equivalent general quartic (biquadratic) algebraic equation. A solution to the general quartic equation following Lagrange's method of symmetric functions yields the desired complex flexural wave numbers in closed form

$$\zeta_1 = \left[ (-a_1 - b_1 + \theta_1 + \theta_2 + \theta_3) / 4 \right]^{1/2}$$

$$\zeta_2 = \left[ (-a_1 - b_1 + \theta_1 - \theta_2 - \theta_3) / 4 \right]^{1/2}$$

$$\zeta_3 = \left[ (-a_1 - b_1 - \theta_1 + \theta_2 - \theta_3) / 4 \right]^{1/2}$$

and

$$\zeta_4 = \left[ (-a_1 - b_1 - \theta_1 - \theta_2 + \theta_3) / 4 \right]^{1/2}$$

$$\text{where } \theta_1 = \left[ y_1 + y_2 - \frac{f}{3} \right]^{1/2}; \quad \theta_2 = \left[ \frac{y_1 + y_2}{2} + i\sqrt{3} \frac{y_1 - y_2}{2} - \frac{f}{3} \right]^{1/2}$$

$$\theta_3 = \left[ \frac{y_1 + y_2}{2} - i\sqrt{3} \frac{y_1 - y_2}{2} - \frac{f}{3} \right]^{1/2}$$

$$y_1 = \left[ -\frac{p}{2} + \left( \frac{p^2}{4} - \frac{q^3}{27} \right)^{1/2} \right]^{1/3}; \quad y_2 = (-q/3y_1)$$

$$p = \frac{1}{27} \left[ + 2f^3 - 9fh - 27k \right]; \quad q = h - \frac{f^2}{3}$$

$$f = 8c_2 - 3c_1^2$$

$$h = 3c_1^4 - 16c_1^2c_2 + 16c_1c_3 + 16c_2^2 - 64c_4$$

$$k = (c_1^3 - 4c_1c_2 + 8c_3)^2$$

$$c_1 = a_1 + b_1; \quad c_2 = a_2 + a_1b_1 + b_2$$

$$c_3 = b_1a_2 + b_2a_1; \quad c_4 = a_2b_2 - \frac{k^{*2}}{E_1I_1E_3I_3}$$

To facilitate comparison with experiment, cantilever end conditions were selected. The equations of constraint for a cantilever beam driven on the free end require zero bending moment at the free end of elastic face layers or

$$\chi_1 = -E_1 I_1 \left( \frac{d^2 w_1}{dx^2} + \frac{\omega^2 \rho_1}{\kappa G_1} w_1 \right) = 0 \quad (4)$$

and a shear force  $S$  equal to the applied concentrated sinusoidal force or

$$S_1 = \kappa b t_1 G_1 \left[ (1 - \sigma_1) \frac{\partial w_1}{\partial x} + \psi_1 \frac{\partial^3 w_1}{\partial x^3} \right] = P_o e^{i\omega t} \quad (5)$$

where

$$\sigma_1 = \frac{\left( \frac{r_1^2 E_1}{\kappa G_1} \right)^2 (n_1)^4 + 1}{\left[ 1 - \frac{n_1^4 r_1^4 E_1}{\kappa G_1} \right]}$$

and

$$\psi_1 = \frac{r_1^2 E_1}{\kappa G \left[ 1 - \frac{n_1^4 r_1^4 E_1}{\kappa G_1} \right]}$$

The boundary conditions on layer 1 at the fixed end constrain motion such that

$$w_1(\ell) = 0 \quad \text{and} \quad \left( \frac{dw_1}{dx} \right)_{x=\ell} = 0 \quad (6)$$

The free-free boundary conditions on the constraining layer (layer 3) required zero bending moment and zero shear force at  $x = 0$  and  $x = \ell$  so that

$$\left. \begin{aligned} \chi_3(0) &= 0 & \chi_3(\ell) &= 0 \\ S_3(0) &= 0 & S_3(\ell) &= 0 \end{aligned} \right\} \quad (7)$$

These eight constraint equations can be placed in matrix form, Equation (9), and the wave amplitude coefficients  $H_i$  can be obtained from inverting the constraint matrix and multiplying by the loading matrix. The quantities  $P_i$  and  $Q_i$  in the constraint matrix are defined by the relations

$$P_n = \kappa b t_1 G_1 \left[ (1 - \sigma_1) \zeta_i + \psi_1 \zeta_i^3 \right]$$

$$Q_n = (-E_1 I_1) \left( \zeta_i^2 + \frac{\omega^2 \rho_1}{\kappa G_1} \right)$$

for  $n = 1$  to 4

and



$$P_n = \kappa b t_3 G_3 \left[ (1 - \sigma_3) \zeta_1 + \psi_3 \zeta_1^3 \right]$$

$$Q_n = (-E_3 I_3) \left( \zeta_1^2 + \frac{\omega^2 \rho_3}{\kappa G_3} \right)$$

for  $n = 5$  to  $8$

(8)

$P_1$	$-P_1$	$P_2$	$-P_2$	$P_3$	$-P_3$	$P_4$	$-P_4$	$H_1$	$P_0$
$Q_1$	$Q_1$	$Q_2$	$Q_2$	$Q_3$	$Q_3$	$Q_4$	$Q_4$	$H_2$	0
$\zeta_1^l$	$-\zeta_1^l$	$\zeta_2^l$	$-\zeta_2^l$	$\zeta_3^l$	$-\zeta_3^l$	$\zeta_4^l$	$-\zeta_4^l$	$H_3$	0
$\zeta_1^e$	$-\zeta_1^e$	$\zeta_2^e$	$-\zeta_2^e$	$\zeta_3^e$	$-\zeta_3^e$	$\zeta_4^e$	$-\zeta_4^e$	$H_4$	0
$P_5 K_1$	$-P_5 K_1$	$P_6 K_2$	$-P_6 K_2$	$P_7 K_3$	$-P_7 K_3$	$P_8 K_4$	$-P_8 K_4$	$H_5$	0
$Q_5 K_1$	$Q_5 K_1$	$Q_6 K_2$	$Q_6 K_2$	$Q_7 K_3$	$Q_7 K_3$	$Q_8 K_4$	$Q_8 K_4$	$H_6$	0
$Q_5 K_1^e$	$Q_5 K_1^e$	$Q_6 K_2^e$	$Q_6 K_2^e$	$Q_7 K_3^e$	$Q_7 K_3^e$	$Q_8 K_4^e$	$Q_8 K_4^e$	$H_7$	0
$P_5 K_1^e$	$-P_5 K_1^e$	$P_6 K_2^e$	$-P_6 K_2^e$	$P_7 K_3^e$	$-P_7 K_3^e$	$P_8 K_4^e$	$-P_8 K_4^e$	$H_8$	0

(9)

The expressions for the mechanical impedance of this system can be calculated from the expressions

$$\begin{aligned}
 Z_1(x, \omega) = \frac{P_0}{i\omega} & \left[ H_1 e^{\zeta_1 x} + H_2 e^{-\zeta_1 x} + H_3 e^{\zeta_2 x} \right. \\
 & \left. + H_4 e^{-\zeta_2 x} + H_5 e^{\zeta_3 x} + H_6 e^{-\zeta_3 x} + H_7 e^{\zeta_4 x} + H_8 e^{-\zeta_4 x} \right]^{-1} \\
 Z_3(x, \omega) = \frac{P_0}{i\omega} & \left[ K_1 \left( H_1 e^{\zeta_1 x} + H_2 e^{-\zeta_1 x} \right) \right. \\
 & + K_2 \left( H_3 e^{\zeta_2 x} + H_4 e^{-\zeta_2 x} \right) \\
 & + K_3 \left( H_5 e^{\zeta_3 x} + H_6 e^{-\zeta_3 x} \right) \\
 & \left. + K_4 \left( H_7 e^{\zeta_4 x} + H_8 e^{-\zeta_4 x} \right) \right]^{-1}
 \end{aligned} \tag{10}$$

Intuitively, one would expect that the composite beam modes with eigenfrequencies in the spectral vicinity of the delamination frequency of the composite would experience the highest compressional damping since the viscoelastic layer receives the greatest dynamic compression strains. Thus, if the elastic layers are treated

as lumped masses and the viscoelastic layer as a distributed complex spring, then this model gives rise to a compressional frequency working to delaminate the face layers at

$$\omega_c^* = \left[ \left( \frac{E_v^*}{t_v} \right) \left( \frac{1}{\rho_1 t_1} + \frac{1}{\rho_3 t_3} \right) \right]^{1/2} = \omega_c' (1 + i\delta)^{1/2} \quad (11)$$

From this result, compressional damping can be characterized as a narrowband phenomenon spectrally centered near  $\omega_c$  with a bandwidth dependent upon the elastic loss tangent of the viscoelastic layer.

#### NEARLY INCOMPRESSIBLE VISCOELASTIC DAMPING CORES

For the case where the viscoelastic damping core is nearly incompressible (i.e.,  $\nu_v^* \rightarrow 1/2$ ), the expression for the compressional frequency of the composite must be suitably modified to account for shape and the complex Poisson's ratio of the core. This modification can take the form of replacing the complex modulus of elasticity  $E_v^*(\omega)$  with the concept of the apparent complex modulus of elasticity  $\tilde{E}_v^*(\omega)$ . An expression for  $\tilde{E}_v^*(\omega)$  can be easily derived from the principle of correspondence for harmonic motion and consideration of Hooke's law applied to an elastic layer sandwiched between two rigid plates such that lateral strain is prevented.

$$\frac{\sigma_z}{\epsilon_z} \equiv \tilde{E}_v^*(\omega) = E_v^* \frac{[1 - \nu_v^*]}{[(1 - 2\nu_v^*)(1 + \nu_v^*)]} \quad (12)$$

From this expression, it is apparent that the compressional frequency of a three-layer damped beam with a thin relatively incompressible viscoelastic layer and low thickness to width ratio will be significantly affected by Poisson's ratio of the core. Figure 1 illustrates the dependence of the compressional frequency of the composite as well as the apparent modulus of elasticity on Poisson's ratio of the

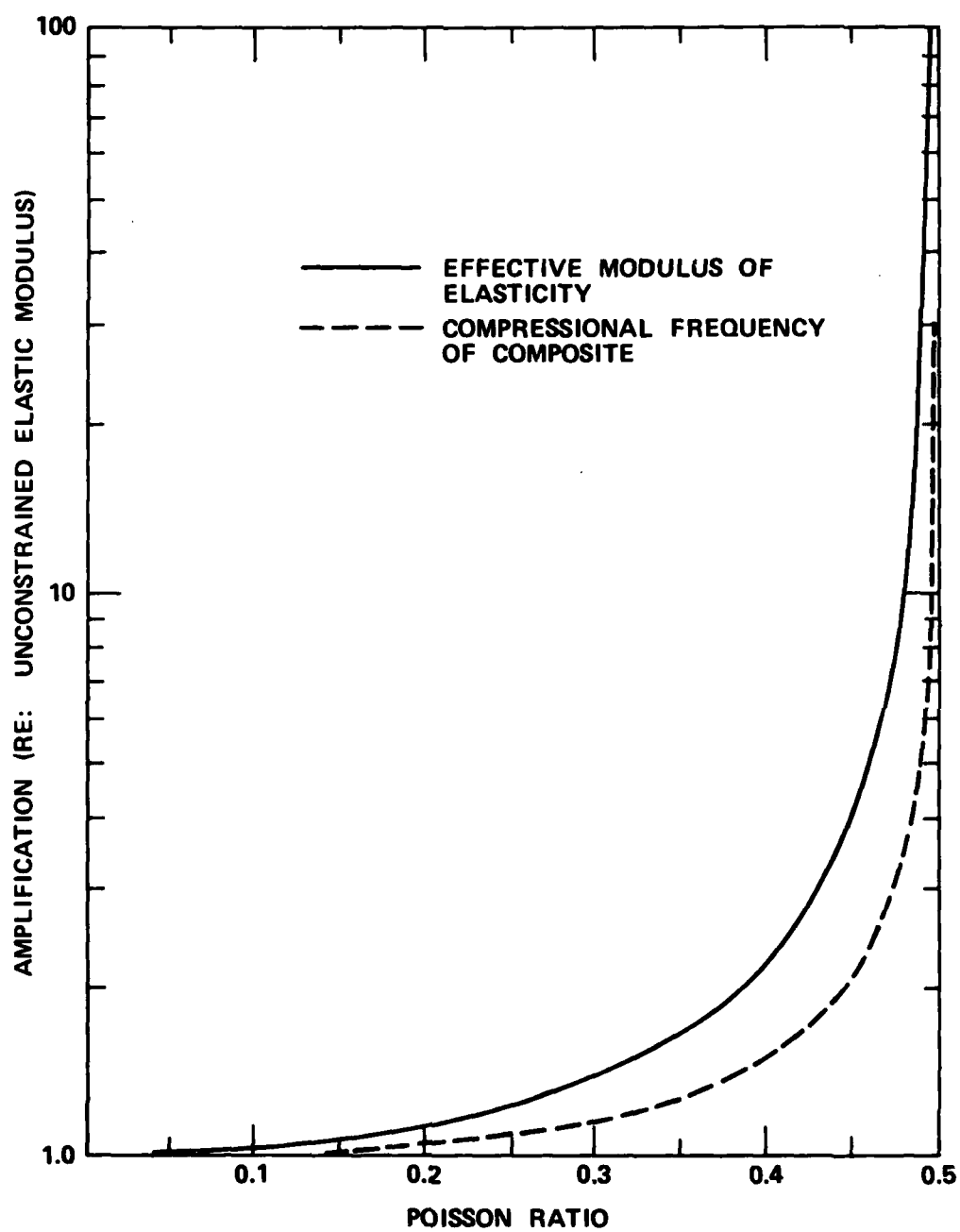


Figure 1 - Effective Modulus of Elasticity as a Function of Poisson's Ratio of the Viscoelastic Core

core assuming that Young's loss tangent is equal to the shear loss tangent (i.e., a real Poisson's ratio). Examination of Figure 1 indicates that the compressional frequency of such a composite is highly sensitive to the value of Poisson's ratio of the core in the range from 0.45 to 0.5 to a degree that such a composite may provide a method of evaluation for Poisson's ratio of viscoelastic materials.

For damped three-layer beams with relatively thick rubberlike damping cores, the apparent complex modulus of elasticity can be written<sup>7-9</sup> for rectangular beams as

$$\tilde{E}_v^*(\omega) = E_v^*(\omega) \left[ \left( 4 + \frac{\beta b^2}{2t_v^2} \right) / 3 \right] \quad (13)$$

where  $\beta$  is a material parameter (e.g.,  $\beta = 2$  for rubber cores unfilled by carbon black).

#### DISCUSSION AND CONCLUSIONS

Using the experimental results reported by Douglas and Yang,<sup>6</sup> a comparison of the analysis presented in this paper with both experiment and a compressional damping model incorporating a Bernoulli-Euler model for the face layers was made in a free mechanical impedance format (Figure 2). The damping core used for this specimen was an acrylic base viscoelastic material whose complex elastic modulus could be functionally approximated at the test temperature (22°C) by the expression

$$E_v = 4.26 \times 10^5 \exp[0.494 \ln(\omega/2\pi)] \text{ N/m}^2 \quad (14)$$

$$\delta = 1.46$$

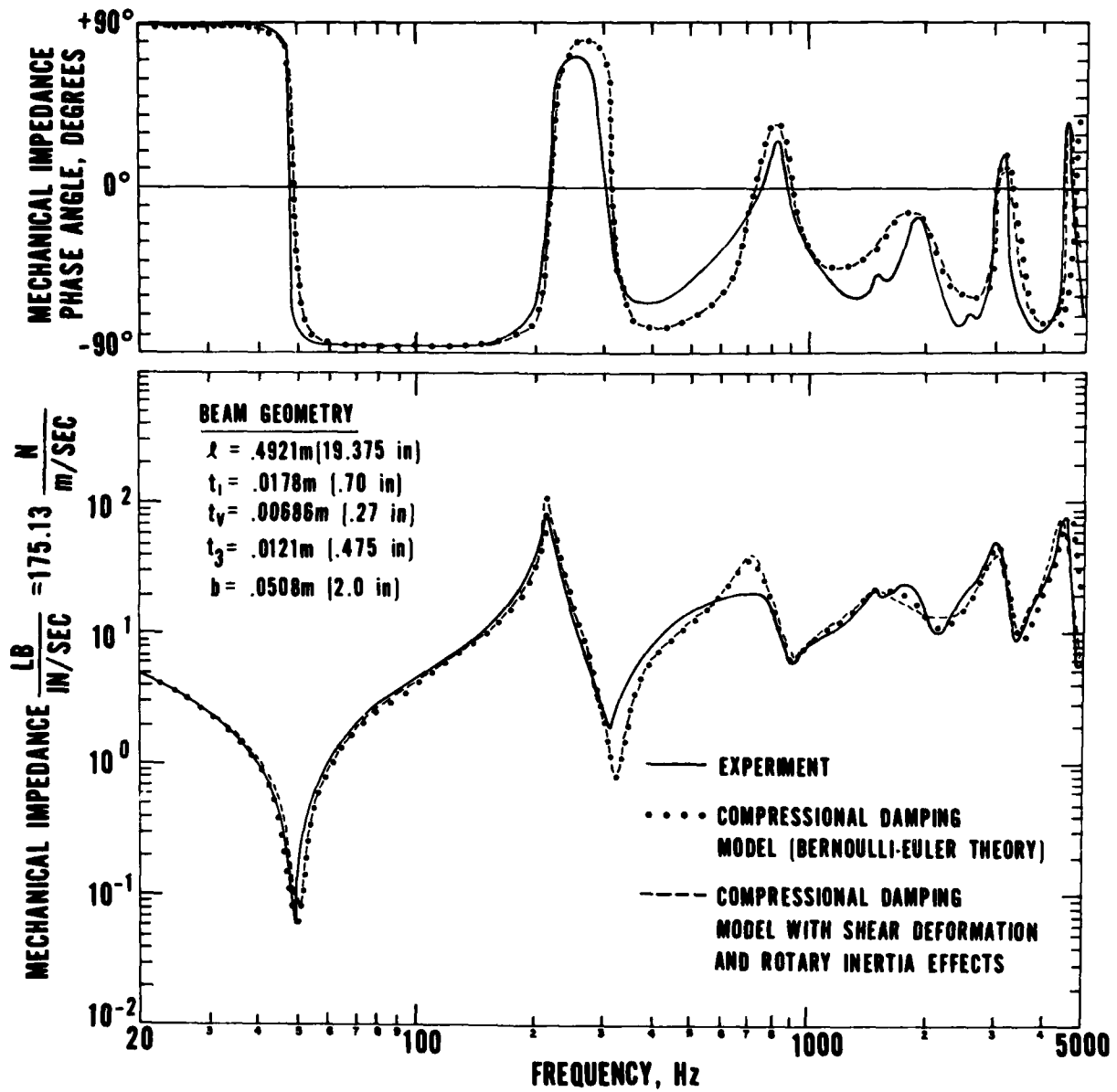


Figure 2 - Transverse Driving Point Free Mechanical Impedance and Phase Angle Spectrum for an Elastic-Viscoelastic-Elastic Beam

Excellent agreement was observed between both analytical models and the experimental results throughout the spectral range of interest. The shear damping model showed a good prediction of the resonance and antiresonance frequencies of the composite but also indicated composite loss factors ranging from less than 0.01 to 0.02, thus indicating that this mechanism is not dominant in the spectral range reported for this specimen. It should be noted that this specimen was intentionally designed to optimize the compressional damping mechanism near 1 kHz and that such a design results in low shear damping.

For most problems where broadband damping is required, such as controlling piping or fan duct vibrations, shear damping remains as the most practical dissipation mechanism to utilize in constrained layer composites. However, for structural response problems where narrowband vibrational energy is the overriding consideration, transverse compressional damping can achieve composite loss factors approaching the elastic (Young's) loss factor of the damping core if suitable attention is paid to Poisson's ratio and shape factor of the core. Unfortunately for most commercially available viscoelastic materials, it is difficult to simultaneously optimize the composite for both shear and compressional damping, especially at low frequencies.

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